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REPORT

**TRAFFIC INFORMATION SERVICE:
the dependence of f.m. capture
effect upon modulation index**

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**TRAFFIC INFORMATION SERVICE: THE DEPENDENCE OF F.M.
CAPTURE EFFECT UPON MODULATION INDEX
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Summary

A theoretical analysis is given of 'capture effect' in f.m., as it applies to the 'ring' system proposed for the traffic information service. It is shown that, in order to exploit the effect to the full, particular values may be chosen for the modulation index that is adopted for the START signal.

Although the theory assumes infinite-bandwidth discriminators in the traffic-information receivers, the results of brief experiments are described which confirm that a discriminator bandwidth approximately three times that of the peak-to-peak deviation adopted for the START signals would provide adequate performance.

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1. Introduction

For the proposed traffic-information service¹ it is considered desirable to adopt some adaptive system, rather than a fixed-threshold system, by which receivers can distinguish between messages from the local transmitter (which they should reproduce) and messages from other, more distant, transmitters (which they should disregard).

An adaptive system which at present shows the greatest promise^{2,3} relies upon the 'capture effect' of f.m. Before radiating its message,* the carrier of the message transmitter is frequency-modulated by a 125 Hz tone for about 0.5 sec. This is termed the START signal. All the surrounding transmitters (termed 'ring' transmitters) are inoperative for the duration of the message but, during this 0.5 sec. period prior to the message, they radiate c.w.** at reduced power. A circuit tuned to 125 Hz in each receiver, subsequent to a carrier-frequency limiter and discriminator, responds to the START signal and, if the amplitude of the demodulated signal is large enough, a latch in the receiver is 'set' so that the subsequent message is reproduced. If the amplitude of the demodulated signal is not large enough the latch is not set and the message is not reproduced. Capture effect in f.m. is such that the amplitude of the demodulated signal at 125 Hz is substantially unaffected by the signal from a ring transmitter if its field strength is less than that of the message transmitter. However, as the field strength of the ring transmitter approaches that of the message transmitter, the amplitude of the demodulated signal at 125 Hz drops sharply and is small when the field strength of the ring transmitter exceeds that of the message transmitter. Where the field strengths of two or more ring transmitters are comparable, the capture effect is less marked, but such areas are small in comparison with the service area of the message transmitter.

Thus capture effect provides a very sensitive indication of the greater of the two field-strengths, independently of their absolute values, and thus reliably defines the service area of the message transmitter.

The nature of capture effect is to some extent dependent on the parameters of the system; given an understanding of this relationship, one can choose the values of the parameters so as to exploit the effect to the full. This Report shows how capture effect depends upon the modulation index adopted for the START signal.

* The type of modulation to be used for the message is immaterial. It is at present envisaged that the message will be amplitude-modulated but frequency-modulation may be found more convenient in the future.

** The carriers of the surrounding transmitters are in fact frequency-modulated at low-deviation by noise during this period in order to avoid beats between the carriers, but, for the purpose of this Report, this may be disregarded.

2. Capture effect in f.m. under idealised conditions

It is shown in the Appendix that, assuming a perfect limiter and discriminator, and a c.w. signal from a single ring transmitter, the audio signal at the discriminator output of each receiver during the START signal comprises the fundamental and odd harmonics of the modulation frequency (125 Hz) adopted for the START signal. If the field strength of the signal from the message transmitter is b times the field strength of the signal from the dominant ring transmitter, the analysis in the Appendix shows that the amplitude of the fundamental component (i.e. at 125 Hz) when $b < 1$, relative to the amplitude of this fundamental components when $b \gg 1$, is given by:

$$A_1 = (2/m) [bJ_1(m) - (b^2/2)J_1(2m) + (b^3/3)J_1(3m) \dots] \quad (1)$$

where m is the modulation index* adopted for the START signal and $J_n(m)$ is the Bessel function of the first kind, of order n and argument m . The analysis also shows that, if $b > 1$, the amplitude of the fundamental component relative to that when $b \gg 1$ is given by:

$$A_2 = 1 - (2/m) [(1/b)J_1(m) - (1/2b^2)J_1(2m) + (1/3b^3)J_1(3m) \dots] \quad (2)$$

In order to optimise the capture effect, it is necessary to make A_1 as small as possible when $b < 1$, and A_2 as nearly unity as possible when $b > 1$. Both these requirements can be met by adopting a value for m such that $J_1(m) = 0$. Since $J_1(m)$ is an oscillatory function an infinite number of values of m may be chosen that would satisfy that requirement. A convenient value, however, is $m \approx 16.47$ because that corresponds to values for peak deviation (2.06 kHz) and modulation frequency (125 Hz) that are convenient in practice. The benefit gained by making this choice is shown in Fig. 1 in which the theoretical levels of the fundamental component of the modulation frequency adopted for the START signal, calculated from Equations (1) and (2), are plotted as a function of b for three values of m . For $m = 16.47$, $J_1(m)$ is zero whereas maxima of $J_1(m)$ occur at $m = 14.86$ and $m = 18.02$. It can be seen that there is an optimum capture effect when $m = 16.47$.

3. The effect of instrumental restrictions

The analysis in Section 2 assumed a perfect limiter and discriminator, together with a perfect filter at the discriminator output. This Section discusses the effect of imperfections in these parts of a traffic-information receiver.

* i.e. the ratio (peak deviation)/(modulation frequency).

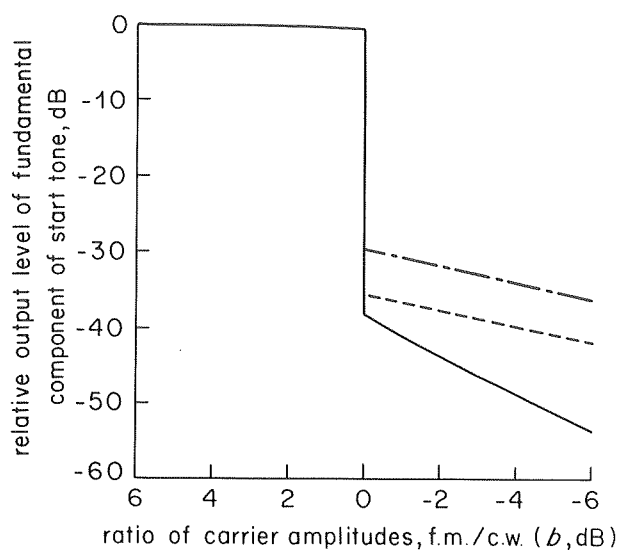


Fig. 1 - Theoretical variation of capture effect as a function of modulation index

- modulation index 14.86
- - - modulation index 16.47
- ... modulation index 18.02

The effect of imperfect limiting, or of bandwidth-restriction in the limiter or discriminator, is to 'round off' the sharp discontinuities that are shown to occur at $b = 1$ (0 dB in Fig. 1).

In the region of capture (i.e. $b \approx 1$) large non-sinusoidal variations in amplitude occur on the resultant signal applied to the limiter. Good limiting is therefore important but, since integrated circuits that provide at least 30 dB a.m. suppression are now commonplace, this feature is readily provided.

As shown in the Appendix, the peak deviation of the resultant signal after limiting is $b/(1-b)$ times the original peak frequency-deviation. If therefore, a well-defined capture effect is to take place for values of b up to 0.75 (i.e. for carrier ratios to within 2.5 dB), the discriminator bandwidth should extend beyond some three times the deviation adopted for the START signal, i.e. beyond ± 6 kHz.

An indication of the importance of discriminator bandwidth is given in Fig. 2 which compares the measured performance of three finite-bandwidth discriminators with the theoretical performance of an infinite-bandwidth discriminator. In Fig. 2, curves (a) and (b) were measured using a comparatively simple limiter, discriminator and low-pass filter combination of the type that is likely to be provided in future traffic-information receivers.⁴ Curve (c), on the other hand, was measured using a high-performance low-pass filter connected at the output of the limiter/discriminator of a deviation meter. The latter was a high-class measuring instrument that was known to incorporate a high-performance limiter and very-wide-bandwidth discriminator.

The three measured results may be taken to indicate the extent to which capture effect depends upon the dis-

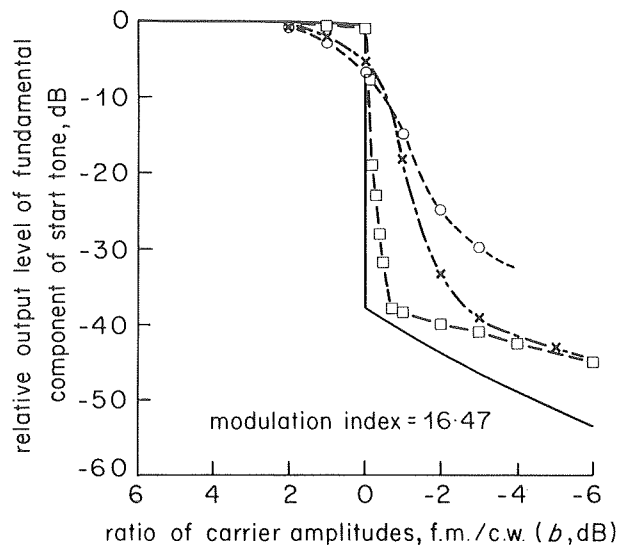


Fig. 2 - Theoretical and measured variation of capture effect as a function of discriminator bandwidth

- (a) ○ - - - ○ Measured: discriminator bandwidth 11 kHz pk-pk
- (b) x - - - x Measured: discriminator bandwidth 40 kHz pk-pk
- (c) □ - - - □ Measured: very-wide-bandwidth discriminator
- (d) — Theoretical: assuming infinite-bandwidth discriminator

criminator bandwidth and upon the performance of the limiter and the low-pass filter.

As shown in the Appendix, the output from the discriminator when $b < 1$ comprises components of the original modulation (START tone) at its fundamental frequency and at odd-harmonic frequencies. The theoretical levels of these components are shown, as a function of b , in Fig. 3. It is evident that, if the full potential capture effect is to be realised, the post-discriminator filter needs to

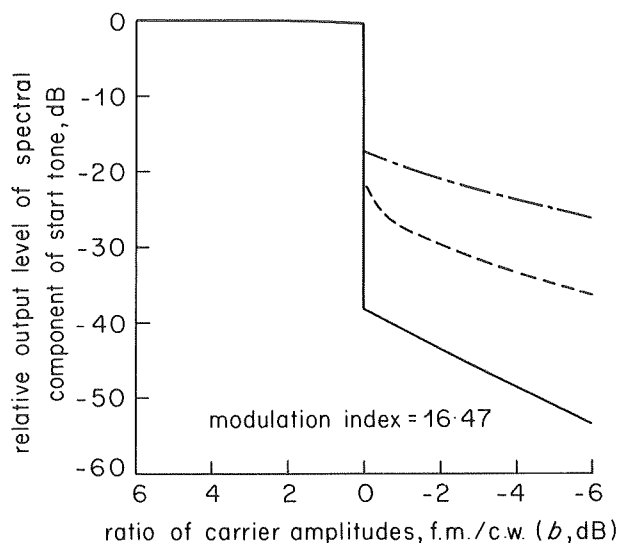


Fig. 3 - Theoretical levels of harmonic components of START tone

- Fundamental component
- - - Third harmonic component
- ... Fifth harmonic component

attenuate all harmonics, relative to the fundamental component, by at least 20 dB. Performance of that order, however, is readily achieved by means of a reasonably simple active filter.

4. Conclusions

This Report has shown that capture effect in f.m., as applied to the proposed traffic-information service, can be optimised by choosing a modulation index (m) for the START signal such that $J_1(m) = 0$.^{*} A convenient value that satisfies this requirement is $m = 16.47$, corresponding to a peak deviation of 2.06 kHz at 125 Hz.

Although the above is a theoretical result assuming an infinite-bandwidth discriminator, it is shown to be in good agreement with measured results.

It is shown that wide discriminator bandwidths are necessary in receivers if the full potential of capture effect

^{*} where $J_n(m)$ is the Bessel function of the first kind, of order n and argument m .

is to be realised. An acceptable performance can be obtained, however, by using a 'peak-to-peak' discriminator bandwidth of about three times the peak-to-peak deviation adopted for the START tone.

5. References

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Appendix

An analysis of 'capture effect' in f.m.

The vector diagram of Fig. 4 represents the vector addition of an unmodulated carrier (OA), of unit amplitude, and a sinusoidally-frequency-modulated carrier (AB) whose rest frequency is that of the unmodulated carrier but whose amplitude is b . The resultant (OB) represents the wave applied to the limiter.

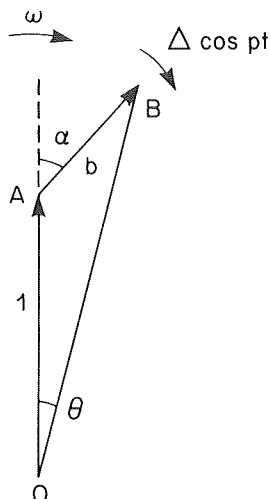


Fig. 4 - Vector addition of f.m. and c.w. signals

The unmodulated carrier may be expressed as $E_1 = \sin \omega t$ and the frequency-modulated carrier as

$$E_2 = b \sin(\omega t + m \cos pt)$$

where $\omega/2\pi$ is the carrier frequency
 $p/2\pi$ is the modulation frequency
 and m is the modulation index
 where $m = \Delta/p$
 and $\Delta/2\pi$ is the peak frequency deviation.

Assuming a perfect limiter and discriminator, the instantaneous amplitude of the output signal from the discriminator is proportional to the instantaneous frequency of the resultant signal, OB (see Fig. 4), applied to it.* If θ and α are the instantaneous phase angles shown in Fig. 4, the instantaneous frequency of the resultant (OB) is given by

$$\omega_i = \omega + d\theta/dt.$$

and, neglecting the d.c. term (if any) arising from ω , the instantaneous amplitude of the output signal from the discriminator is given by

$$e = kd\theta/dt \quad (3)$$

* This quasi-stationary-state solution is valid here because the limiter and discriminator are assumed to impose no bandwidth restriction.

where k is an arbitrary constant.

Now

$$\tan \theta = b \sin \alpha / (1 + b \cos \alpha)$$

where

$$\alpha = (\Delta/p) \sin pt$$

hence

$$\begin{aligned} d\theta/dt &= (d\alpha/dt) \cdot b(b + \cos \alpha) / (1 + 2b \cos \alpha + b^2) \\ &= (\Delta \cos pt) b(b + \cos \alpha) / (1 + 2b \cos \alpha + b^2) \end{aligned} \quad (4)$$

$$\text{Now } e^{i\alpha} / (1 + be^{i\alpha}) = (b + \cos \alpha + i \sin \alpha) / (1 + 2b \cos \alpha + b^2)$$

where

$$i = \sqrt{-1}$$

$$\text{Thus } \text{Re } e^{i\alpha} / (1 + be^{i\alpha}) = (b + \cos \alpha) / (1 + 2b \cos \alpha + b^2) \quad (5)$$

where R means 'the real part of'.

If $b < 1$, we may expand Equation (5) as a convergent series by the Binomial theorem:

$$\text{Re } e^{i\alpha} / (1 + be^{i\alpha}) = \text{Re } e^{i\alpha} (1 - be^{i\alpha} + b^2 e^{i2\alpha} - b^3 e^{i3\alpha} + \dots) \quad (6)$$

Hence, combining Equations (3), (4), (5) and (6):

$$\begin{aligned} e &= k\Delta \cos pt \cdot b(b + \cos \alpha) / (1 + 2b \cos \alpha + b^2) \\ &= Rk\Delta \cos pt \cdot (be^{i\alpha} - b^2 e^{i2\alpha} + b^3 e^{i3\alpha} - b^4 e^{i4\alpha} + \dots) \\ &= k\Delta \cos pt \cdot \sum_{n=1}^{\infty} (-1)^{n+1} b^n \cos n\alpha \\ &= k\Delta \cos pt \cdot \sum_{n=1}^{\infty} (-1)^{n+1} b^n \cos (nm \sin pt) \end{aligned} \quad (7)$$

$$\text{Now } \cos(nm \sin pt) = J_0(nm) + 2 \sum_{r=1}^{\infty} J_{2r}(nm) \cos 2rpt \quad (8)$$

where $J_r(nm)$ is the Bessel function of the first kind, of order r and argument (nm) . Furthermore, since the expansions given in Equations (7) and (8) are absolutely convergent, we may re-group their terms in any order. Hence, combining Equations (7) and (8) and arranging their terms as a series of harmonics of p ,

$$e = k\Delta \left\{ \begin{aligned} &(bJ_0(m) - b^2 J_0(2m) + b^3 J_0(3m) \dots) \cos pt \\ &+ 2(bJ_2(m) - b^2 J_2(2m) + b^3 J_2(3m) \dots) \cos pt \cos 2pt \\ &+ 2(bJ_4(m) - b^2 J_4(2m) + b^3 J_4(3m) \dots) \cos pt \cos 4pt \\ &+ \dots \end{aligned} \right\}$$

$$= k\Delta \left\{ \begin{aligned} & \sum_{n=1}^{\infty} (-1)^{n+1} b^n [J_0(nm) + J_2(nm)] \cos pt \\ & + \sum_{n=1}^{\infty} (-1)^{n+1} b^n [J_2(nm) + J_4(nm)] \cos 3pt \\ & + \sum_{n=1}^{\infty} (-1)^{n+1} b^n [J_4(nm) + J_6(nm)] \cos 5pt \\ & + \dots \end{aligned} \right\}$$

Since $J_{(r-1)}(nm) + J_{(r+1)}(nm) = (2r/nm) J_r(nm)$

Equation (9) may be re-written as

$$e = (2k\Delta/m) \sum_{r=1}^{\infty} \sum_{n=1}^{\infty} (-1)^{n+1} (b^n/n) J_r(nm) \cos rpt \quad (10)$$

where r is given odd integral values 1, 3, 5, 7, etc.

Without the unmodulated carrier, the amplitude of the output signal would be:

$$e = k\Delta$$

Thus, as given in Equation (1) of Section 2, the amplitude of the fundamental component of the output signal in the presence of the unmodulated carrier, relative to that without the carrier, is given by:

$$A_1 = (2/m) \left[\sum_{n=1}^{\infty} (-1)^{n+1} (b^n/n) J_1(nm) \right]$$

The foregoing has assumed that $b < 1$. If $b > 1$ we may re-write Equation (4) as:

$$d\theta/dt = (\Delta \cos pt)(1 + x \cos \alpha)/(1 + 2x \cos \alpha + x^2) \quad (11)$$

where $x = 1/b$

$$\text{Now } R(1 + xe^{i\alpha})^{-1} = (1 + x \cos \alpha)/(1 + 2x \cos \alpha + x^2) \quad (12)$$

where, as for Equation (5), R means 'real part of' and $i = \sqrt{-1}$.

Thus, if $x < 1$, we may expand Equation (12) by the Binomial theorem:

$$\begin{aligned} (1 + x \cos \alpha)/(1 + 2x \cos \alpha + x^2) &= \\ &= R(1 - xe^{i\alpha} + x^2 e^{i2\alpha} - x^3 e^{i3\alpha} + \dots) \\ &= 1 - \sum_{n=1}^{\infty} (-1)^{n+1} x^n \cos n\alpha \end{aligned} \quad (13)$$

Combining Equations (3), (11), (12) and (13), we have:

$$e = (k\Delta \cos pt) \left[1 - \sum_{n=1}^{\infty} (-1)^{n+1} b^{-n} \cos(nm \sin pt) \right] \quad (14)$$

By expanding Equation (14) in terms of Bessel functions, as for Equation (7), it may readily be shown that, when $b > 1$, the amplitude of the r th harmonic of p , relative to the amplitude of the fundamental in the absence of the carrier, is given by:

$$A_r = 1 - (2r/m) \left[\sum_{n=1}^{\infty} (-1)^{n+1} (1/nb^n) J_r(nm) \right]$$

where, as for Equation (10), r is given odd integral values 1, 3, 5, 7, etc. With $r = 1$, this expression was given as Equation (2) in Section 2.

The instantaneous frequency deviation of the signal at the limiter output is given by Equation (4). This has its maximum value when $\alpha = (2n - 1)\pi/2$ and $pt = n\pi/2$. This maximum value is given by:

$$\begin{aligned} d\theta/dt_{(\max)} &= \frac{\Delta b(b - 1)}{1 - 2b + b^2} \\ &= \frac{\Delta b}{1 - b} \text{ provided } b \neq 1 \end{aligned}$$

This was the expression quoted in Section 3.

